

## **MATHEMATICAL MODELING OF HEAT AND MASS TRANSFER IN EVAPORATION OF A SUBSTANCE FROM A POROUS BODY IN VACUUM**

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*A problem of the Stefan type which describes heat and mass transfer in a porous body with the double phase transition liquid-ice-vapor has been formulated and solved.*

**Introduction.** Investigations of heat and mass transfer processes with phase transitions in unsaturated porous media are of importance for study of moisture motion in soils that is accompanied by complex physicochemical transformations, for development of technological processes of drying of moist materials, including sublimation drying [1, 2], and for evaporative cooling. These investigations are based on either the A. V. Luikov theory of interconnected heat and moisture transfer [1–4] in which the potentials of heat transfer (temperature) and moisture transfer (dependent on the form of the bond of the moisture with the material) and corresponding transfer coefficients are introduced or on the theory of multiphase filtration [5–7].

During the period of a decreasing rate of drying, deepening of the evaporation zone into the body is observed. If the removal of a vapor is so intense that the capillary mechanism of transfer does not ensure the feed of a liquid to dried pores, a front appears which moves into the body depth. In the general case, evaporation occurs not only on the deepened surface but in a certain zone as well, which is attributed, in particular, to different forms of the bond of moisture with the material. This is also true of sublimation drying [1, 2], i.e., drying of material in the frozen state. Previous freezing or self-freezing (due to intense evaporation in vacuum) is used depending on the technology of drying.

One efficient heat-protection method which is widely used in space technology is evaporative cooling [8–11] wherein a heat-transfer agent is evaporated (or sublimated) in a porous wall. In the case of a liquid heat-transfer agent in a porous body, three zones of motion of the heat-transfer agent are present, as a rule: liquid, two-phase, and vapor ones. A modification of porous cooling is the use of self-cooling in devices for which one operating condition is their high erosion resistance. For example, it is possible to attain a decrease in the temperature of the heat-affected surface by filling or impregnating porous tungsten with other material [4, 12, 13].

Different formulations of the problems of heat and mass transfer with phase transformations in capillary-porous bodies are discussed, for example, in [1–5, 7, 8]. In particular, problems with a moving boundary of the evaporation (sublimation) front deepened in a porous body, as a rule, are not concerned with the mass-transfer equation in the evaporation zone. These problems are reduced to the Stefan problems. Their characteristic feature is that the temperature on this boundary is generally a quantity which is variable with time and unknown prior to solution [4, 13]. Therefore, a correct mathematical formulation of such problems necessitates an additional relation connecting the velocity of motion of the evaporation front and the front temperature. Analogous relations can be obtained in specific cases for model porous media from consideration of the kinetics of mass transfer in individual capillaries [4] (one such condition is used in this work). Actually, the temperature on the evaporation boundary is often considered to be a quantity which is either known in advance or determined by solution of the problem with the use of the law of motion of the evaporation boundary found experimentally. For example, in [14] the exact solution of a problem of the Stefan type on sublimation in a porous half-space has been obtained under the assumption that a certain constant sublimation temperature is specified on the moving front; in [15], the temperature of the sublimation front is taken to be

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unknown but constant. The solutions of the equations of vapor diffusion have also been obtained in both works. In [16], unlike [14, 15], it is assumed that vapor flow in the dried region of a porous medium occurs as a result of the presence of the gradients of both moisture concentration and pressure. It has been shown that in the case of sublimation drying the Darcy law is the dominant driving force. It should be noted that this statement is most likely to be true for vapor outflow in a nearly continuum regime.

Investigation of the mechanisms of the processes of sublimation of ice in different disperse systems has been the focus of [17]. In this monograph, consideration has been given to the features of heat and mass transfer in sublimation in disperse systems, and the influence of the structure and physicochemical properties of rocks on the sublimation process has been described. In [18], the features of the transfer processes in sublimation drying and in sublimation of ice from a permeable plate into vacuum with different methods of heat supply have been presented and problems related to sublimation kinetics have been discussed. Based on the experimental investigations performed, it has been shown that three zones generally arise in the permeable plate in the process of sublimation of ice: zones of water, ice, and steam. A change in the zone dimensions and the temperature fields is of a pulsating character; the pulsation cycle can be subdivided into two periods of time: microbreak and rise of the sublimation surface (ice plug) and freezing and descent of the sublimation surface. However the thickness of the ice layer is very small and the temperature of the evaporation surface is close to  $0^{\circ}\text{C}$  for cermet plates with fine pores; pulsations are absent, in practice, and the sublimation (evaporation) surface occupies a constant position in the plate. In this work, we have also proposed a formulation of the problem of heat and mass transfer with three phase regions.

The liquid–ice–vapor double phase transition also exists with operation of a sublimation heat exchanger in outer space. This fact has been described in [19], where it is argued that when water is supplied to a sublimation gap it penetrates into the pores and freezes on the boundary with vacuum, forming ice plugs which retard the water ejection; this is the basic advantage of the sublimator heat exchanger. Under the influence of heat incoming through the partition, the sublimation gap, and the skeleton of the porous element from the cooled heat-transfer agent, sublimation into vacuum occurs. As time passes, the ice–steam interphase moves more deeply into the porous body. The thickness of the ice layer gradually decreases, so that at a certain instant, under the influence of pressure in the sublimation gap, the liquid breaks the ice and freezes, reaching the external boundary of the porous element. Later on, the process is repeated.

It should be noted that in [18] other hypotheses for moisture motion, coexistence of three phases in a porous body, and cyclicity of the process of sublimation cooling are also discussed. In particular, it is argued that the formation of finely crystalline ice, the destruction of it near the walls of the porous-body skeleton (in the region of energy supply), and the ejection (break) of a liquid near the skeleton walls under the influence of the pressure gradient occur. Furthermore, as has been shown in [20], nonfreezing thin liquid fillers between the ice and the solid hydrophilic substrate are preserved under the influence of surface forces at a temperature below the triple point, and the thermocrystallization mass flux also occurs in the porous body in the presence of the temperature gradient.

**Formulation of the Problem.** We consider a porous material representing a porous cylinder of nickel (Fig. 1). A liquid is fed to the sublimation gap ( $0 < r < L$ ) under pressure; then it moves under the influence of filtration and capillary pressures through the internal boundary  $r = L$  of the porous element to the external surface of the cylinder, which is in contact with vacuum. As a result of the evaporation of the liquid and the radiative removal of heat, the porous element is cooled and the liquid freezes, forming an ice filler in the pores.

Based on the assumption of the identity of the boundary conditions along the cylinder length, we will consider the process of heat and mass transfer in the porous cylinder to be one-dimensional. A heat flux is supplied to the internal surface  $r = L$ , which, passing through the porous wall filled with liquid and ice, is partially consumed by melting and sublimation (evaporation) and in part is removed from the surface of the material to vacuum by radiation. Thus, following the overall picture of the heat and mass transfer process which is presented in the Introduction, we consider that three phases and the ice–vapor and ice–liquid phase transformations whose boundaries  $r = \xi_1(\tau)$  and  $r = \xi_2(\tau)$  are in motion, coexist in the porous element.

Let us consider a system of differential equations describing a problem of the Stefan type with two fronts of phase transitions. We also take into account that the law of conservation of mass holds, i.e., the quantity of the liquid supplied to the system is equal to the vapor flux removed to vacuum.

Then we have

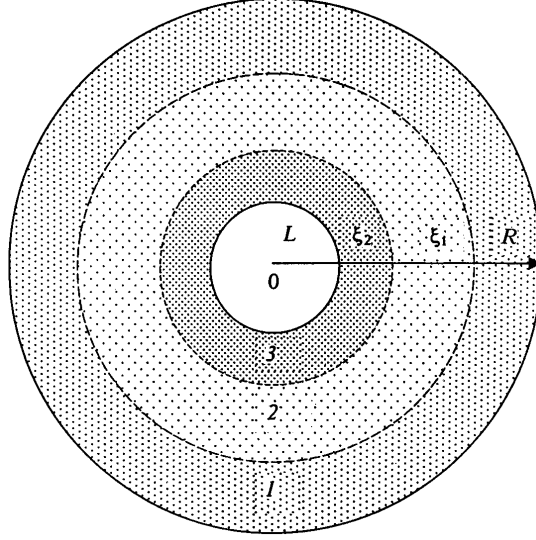


Fig. 1. Geometric scheme of the problem: 1, 2, and 3 correspond to the steam, ice, and water zones in the porous element.

$$(1 - \varepsilon) (\rho c)_s \frac{\partial T_1}{\partial \tau} = \lambda_{\text{eff1}} \left( \frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} \right); \quad \xi_1(\tau) < r < R; \quad (1)$$

$$[\varepsilon(\rho c)_{\text{ice}} + (1 - \varepsilon) (\rho c)_s] \frac{\partial T_2}{\partial \tau} = \lambda_{\text{eff2}} \left( \frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r} \frac{\partial T_2}{\partial r} \right); \quad \xi_2(\tau) < r < \xi_1(\tau); \quad (2)$$

$$[\varepsilon (\rho c)_w + (1 - \varepsilon) (\rho c)_s] \frac{\partial T_3}{\partial \tau} = \lambda_{\text{eff2}} \left( \frac{\partial^2 T_3}{\partial r^2} + \frac{1}{r} \frac{\partial T_3}{\partial r} \right); \quad L < r < \xi_2(\tau). \quad (3)$$

The boundary conditions are as follows:

$$-\lambda_{\text{eff2}} \frac{\partial T_2}{\partial r} \Big|_{\xi_1} + \lambda_{\text{eff1}} \frac{\partial T_1}{\partial r} \Big|_{\xi_1} = -\varepsilon \rho_{\text{ice}} Q_1 \frac{d\xi_1}{d\tau}; \quad \left( \frac{d\xi_1}{d\tau} < 0 \right); \quad (4)$$

$$T_1 \Big|_{r=\xi_1} = T_2 \Big|_{r=\xi_2} = T_*(\tau), \quad (5)$$

$$\rho_{\text{ice}} \frac{d\xi_1}{d\tau} = \frac{\rho_v(T_*) \sqrt{kT_*/2\pi m}}{1 + \xi_1/d} \equiv j_v, \quad (6)$$

$$-\lambda_{\text{eff3}} \frac{\partial T_3}{\partial r} \Big|_{\xi_2} + \lambda_{\text{eff2}} \frac{\partial T_2}{\partial r} \Big|_{\xi_2} = \varepsilon \rho_w Q_2 \frac{d\xi_2}{d\tau}, \quad (7)$$

$$T_2 \Big|_{r=\xi_2} = T_3 \Big|_{r=\xi_2} = T_{\text{ph}}, \quad (T_{\text{ph}} = 273 \text{ K, i. e., } 0^\circ\text{C}), \quad (8)$$

$$-\lambda_{\text{eff3}} \left. \frac{\partial T_3}{\partial r} \right|_L = q. \quad (9)$$

Relations (4) and (5) are the Stefan conditions for the ice–steam phase transition (ice sublimation); the temperature on the phase surface is unknown and must be determined in the course of solution of the problem. Expressions (7) and (8) are the Stefan conditions for the water–ice phase transition (crystallization, melting); however, here the temperature on the boundary of phase transition is known and is equal to  $T_{\text{ph}}$ .

With the deepened front of sublimation  $r = \xi_1(\tau)$ , we have heat removal from the external surface of the porous material by radiation, which enables us to write the boundary condition at  $r = R$  as

$$-\lambda_{\text{eff1}} \left. \frac{\partial T_1}{\partial r} \right|_R = (1 - \varepsilon) \varepsilon^* \sigma T^A \Big|_R. \quad (10')$$

When the porous element is filled with liquid and the liquid evaporates from the surface, the boundary condition has the following form:

$$-\lambda_{\text{eff3}} \left. \frac{\partial T_3}{\partial r} \right|_R = (1 - \varepsilon) \varepsilon^* \sigma T^A \Big|_R + \varepsilon \rho_v(T_R) Q_3 \sqrt{\frac{kT \Big|_R}{2\pi m}}. \quad (10'')$$

As has been noted above, the liquid evaporating from the surface is replaced by the liquid from the internal layers of the porous element. This results in the freezing of water on the external surface of the material without the deepening of the front inward. When water freezes, the sublimation of ice from the surface occurs and we can write the following boundary condition:

$$-\lambda_{\text{eff2}} \left. \frac{\partial T_2}{\partial r} \right|_R = (1 - \varepsilon) \varepsilon^* \sigma T^A \Big|_R + \varepsilon \rho_v(T_R) Q_1 \sqrt{\frac{kT \Big|_R}{2\pi m}}. \quad (10''')$$

The initial conditions have the form

$$\tau = 0, \quad \xi_1(0) = \xi_2(0) = 0, \quad T = 274 \text{ K}. \quad (11)$$

Let us dwell on boundary condition (6) in more detail. It replaces the equation of transfer of the vapor mass in zone 1, relates the unknown temperature  $T_*$  to the sublimation rate, and accounts for the presence of resistance to vapor motion in deepening of the sublimation front. Expression (6) has been obtained as a result of solution of the kinetic equation of vapor transfer in the free-molecular regime in a capillary of finite length in evaporation of the filler in it [4]. As has been shown in [4], this expression can be used with good accuracy for rarefactions from the free-molecular regime to the flow at  $\text{Kn} \approx 1$  (here  $\text{Kn} \approx 2\lambda^*/d^*$ ). Furthermore, with the above flow regimes, expression (6) can also be used for the case of the considered globular structure of the porous element where  $d$  is interpreted as the diameter of a particle of a porous body. Such a conclusion can be drawn, in particular, based on a comparison of (6) with the analogous formula for the vapor flux from a porous layer of globular structure in the free-molecular regime (true, for a highly porous body) [4]:

$$j_v = \frac{\varepsilon \rho_v(T_*) \sqrt{kT_*/2\pi m}}{1 + \frac{9}{8} \frac{\xi_1(1 - \varepsilon)}{d}}.$$

Consequently, boundary-value problem (1)–(11) is a closed problem describing the temperature distributions in the three zones with account for the phase transitions and the presence of moving interphases. The problem is substantially nonlinear, which is why it is solved by a numerical method.

**Procedure of Numerical Solution.** The equations have been approximated following the control-volume method. The possible nonregularity of the spatial grid and variability of the thermophysical characteristics have been taken into account. Then the common difference equation for the three zones is written as

$$c_{\text{eff}} \frac{T_i^k - T_i^{k-1}}{\Delta\tau} = \frac{1}{r_i h_i} \left( r_{i+1/2} \frac{\lambda_{\text{eff}i+1/2}}{\Delta r_i^+} (T_{i+1}^{k-1} - T_i^{k-1}) - r_{i-1/2} \frac{\lambda_{\text{eff}i-1/2}}{\Delta r_i^-} (T_i^{k-1} - T_{i-1}^{k-1}) \right). \quad (12)$$

The coefficients  $c_{\text{eff}}$  and  $\lambda_{\text{eff}i}$  are calculated with account for the distinctive features of the zones and the presence of the interphases. For this purpose we have created special procedures which enable us to carry out the given operations. As is clear from formula (12), the difference scheme is explicit. The final expression for calculation of the temperature has the form

$$T_i^k = T_i^{k-1} + \frac{b}{a} (T_{i+1}^{k-1} - T_i^{k-1}) - \frac{\bar{c}}{a} (T_i^{k-1} - T_{i-1}^{k-1}), \quad (13)$$

where  $a = c_{\text{eff}}/\Delta\tau$ ;

$$c_{\text{eff}} = \begin{cases} (1 - \varepsilon) (\rho c)_s, & \xi_1 < r < R, \\ \varepsilon (\rho c)_{\text{ice}} + (1 - \varepsilon) (\rho c)_s, & \xi_2 < r < \xi_1, \\ \varepsilon (\rho c)_w + (1 - \varepsilon) (\rho c)_s, & L < r < \xi_2 (\tau); \end{cases}$$

$$b = \lambda_{\text{eff}i+1/2} \frac{r_{i+1/2}}{r_i h_i \Delta r_i^+}; \quad \bar{c} = \lambda_{\text{eff}i-1/2} \frac{r_{i-1/2}}{r_i h_i \Delta r_i^-}; \quad h = r_{i+1/2} - r_{i-1/2}; \quad r_{i+1/2} = \frac{r_{i+1} + r_i}{2};$$

$$r_{i-1/2} = \frac{r_1 + r_{i-1}}{2}; \quad \Delta r_i^+ = r_{i+1} - r_i; \quad \Delta r_i^- = r_1 - r_{i-1}.$$

The temperature at points of the porous element is calculated according to Eq. (13). We would like to note that the effect of volume increase in the water–ice phase transition was not taken into account for simplicity of the calculations.

It is necessary to know the numerical values of  $\rho_v$  in the equations considered. For this purpose we use the equation of state of an ideal gas. Here we assume that the vapor is in dynamic equilibrium with the solid or liquid phase. The saturated-vapor pressure as a function of the temperature has been found from the Clausius–Clapeyron formula

$$P_v = P_0 \exp \left( \frac{Q}{R^*} \frac{(T - T_0)}{(T + T_0)} \right). \quad (14)$$

Based on formula (14), we have obtained the expression for the vapor which is in equilibrium with ice

$$P_v = 1.08 \exp \left( 28.84 \frac{T - 213}{T} \right), \quad (15)$$

and for the vapor which is in equilibrium with water we have

$$P_v = 872 \exp \left( 19.47 \frac{T - 278}{T} \right). \quad (16)$$

These formulas adequately describe the steam pressure in the temperature range of interest and their errors do not exceed 9%.

Based on Eqs. (4) and (6) we have composed the difference scheme for numerical calculations

$$T_i = \frac{a_1}{a_1 + b_1} T_{i-1} + \frac{b_1}{a_1 + b_1} T_{i+1} - \frac{\varepsilon \rho_v Q_1 \sqrt{\frac{kT_i}{2\pi m}}}{(a_1 + b_1) \left(1 + \frac{\xi_1}{d}\right)}, \quad (17)$$

where  $a_1 = \frac{\lambda_{\text{eff}i-1/2}}{\Delta r_i^-}$  and  $b_1 = \frac{\lambda_{\text{eff}i+1/2}}{\Delta r_i^+}$ .

Let us find  $T_i$ , solving (17) by the iteration method. Then we determine the mass of sublimated ice from the formula for  $j_v$ , which enables us to calculate the location and velocity of the sublimation front. To do this we have composed the corresponding procedures and algorithms of such a calculation. When the entire ice in the  $i$ th element is sublimated, the sublimation front moves inward. The reverse motion of the sublimation front (unlike the crystallization front) is absent.

From Eq. (7) we have obtained the following difference relation:

$$T_i = \frac{a_1}{a_1 + b_1} T_{i-1} + \frac{b_1}{a_1 + b_1} T_{i+1} + \frac{Q_2 \Delta M}{2\pi r_i H \Delta \tau}. \quad (18)$$

When the entire liquid in the  $i$ th element transforms to ice, we carry out a change of the state of aggregation of the element and shift the zone of phase transition deep into the material. The analogous operations are also performed with the front motion to the external surface in melting of ice.

We obtain from (18) the mass of liquid transformed into ice and vice versa. Based on this, we calculate the location and velocity of the phase front. We have composed the procedure and algorithms of such a calculation which enable us to calculate both the crystallization of water and the melting of ice depending on the cooling or heating of the liquid.

Using expressions (10) and the control-volume method, we have written the difference equation on the external boundary of the material which allows us to account for the cooling of the material by radiation:

$$T_i^k = T_i^{k-1} - \frac{b_2}{a} (T_i^{k-1})^4 - \frac{\bar{c}}{a} (T_i^{k-1} - T_{i-1}^{k-1}), \quad (19)$$

where

$$a = c_{\text{eff}} / \Delta \tau; \quad b_2 = \frac{\varepsilon^* \sigma (1 - \varepsilon)}{h_i}; \quad \bar{c} = \lambda_{\text{eff}i-1/2} \frac{r_{i-1/2}}{r_i h_i \Delta r_i^-}.$$

If there is evaporation of the liquid from the surface, on the basis of Eq. (10'') we have

$$T_i = T_{i-1} - \frac{(1 - \varepsilon) \varepsilon^* \sigma}{a} (T_i)^4 - \frac{\varepsilon \rho_v Q_3}{a} \sqrt{\frac{kT_i}{2\pi m}}, \quad a = \frac{\lambda_{\text{eff}i-1/2}}{\Delta r_i^-}. \quad (20)$$

The analogous equation is obtained in the case where ice is sublimated from the surface  $r = R$  (but  $Q_3$  should be substituted for  $Q_1$ ).

For the internal surface of the cylinder we obtain the following difference relation, using Eq. (9) and the control-volume method:

$$T_i^k = T_i^{k-1} + \frac{b}{a} (T_{i+1}^{k-1} - T_i^{k-1}) + \frac{1}{ah_i} q, \quad (21)$$

where

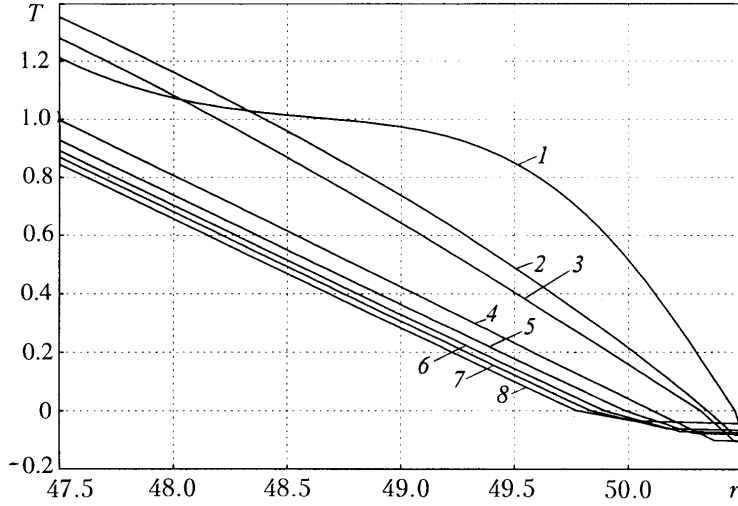


Fig. 2. Temperature distribution over the thickness of the porous element for different instants of time, including crystallization on the water–ice front, at  $\bar{Q} = 250$  W: 1)  $\tau = 0.1$ , 2) 0.5, 3) 1, 4) 5, 5) 10, 6) 15, 7) 20, and 8) 38 sec.  $T$ , °C;  $r$ , mm.

$$a = c_{\text{eff}}/\Delta\tau; \quad b = \lambda_{\text{eff}i+1/2} \frac{r_{i+1/2}}{r_i h_i \Delta r_i^+}.$$

**Discussion of the Results Obtained.** Based on the elaborated procedure, we have created algorithms and programs allowing us to calculate the temperature fields in the porous element, the heat fluxes lost to vacuum by a wick, and the state of aggregation of water in the material. Varying the heat regimes and the thermophysical characteristics of the porous element and its geometric parameters, we have carried out computational experiments on determination of the quantities investigated.

In the calculations, we have used the following data;  $\varepsilon = 0.4$ ,  $\rho_s = 8800$  kg/m<sup>3</sup>,  $\rho_w = 1000$  kg/m<sup>3</sup>,  $\rho_{\text{ice}} = 917$  kg/m<sup>3</sup>,  $c_s = 460$  J/(kg·K),  $c_{\text{ice}} = 2100$  J/(kg·K),  $c_w = 4200$  J/(kg·K),  $\lambda_s = 58$  W/(m·K),  $\lambda_{\text{ice}} = 2.21$  W/(m·K), and  $\lambda_w = 0.55$  W/(m·K);  $\lambda_{\text{eff}1} = 5$  W/(m·K) is the effective thermal conductivity of porous nickel filled with vapor [18],  $\lambda_{\text{eff}2} = 25$  W/(m·K) is the same with ice filling [18], and  $\lambda_{\text{eff}3} = 10$  W/(m·K) is the same with water filling,  $k = 1.38 \cdot 10^{-23}$  J/K,  $m = 2.99 \cdot 10^{-23}$  kg,  $R^* = 461.9$  J/(kg·K),  $Q_1 = 2.83 \cdot 10^6$  J/kg,  $Q_2 = 3.34 \cdot 10^5$  J/kg,  $Q_3 = 2.5 \cdot 10^6$  J/kg,  $d = 35 \cdot 10^{-6}$  m,  $L = 0.0475$  m,  $D = 0.003$  m,  $\varepsilon^* = 0.3$ ,  $\sigma = 5.67 \cdot 10^{-8}$  W/(m<sup>2</sup>·K<sup>4</sup>),  $H = 0.18$  m, and  $q = 1523\text{--}15,230$  W/m<sup>2</sup> (which corresponds to a heat power of  $\bar{Q} = 100\text{--}1000$  W).

Figure 2 shows a representative temperature distribution along the depth of the porous element at different instants of time. Three zones separated by two kink points are seen. These points are the surfaces of the ice–steam and water–ice phase transitions. The first zone (see Fig. 1) is a porous structure filled with steam. The second zone is filled with ice and the third one with water.

On the basis of numerical experiments, we established the following representative picture of the process of cooling. At the initial instant, it is assumed that the porous plate is totally filled with water and the liquid evaporates from its surface into vacuum. It is considered that as the moisture evaporates it has a chance to come to the surface from the internal layers of the element due to capillary forces and to the pressure drop maintained in the system. As a result, the surface will be cooled very intensely and at a certain instant the upper layer will freeze, having reached 0°C. Water turns out to be separated from vacuum by an ice layer. From this instant, ice sublimation begins and the sublimation front gradually bites more deeply into the porous element. Simultaneously, a front of the crystallization of the liquid into ice is formed. This front, as shown by numerical calculations, due to sublimation cooling will move faster than the sublimation front. The ice filler will increase. This continues as long as the flux of the arriving heat is smaller than the heat flux removed to vacuum. However the resistance of the porous material increases as a result of the deepening of the sublimation front, which leads to a decrease in the sublimation intensity. At a certain instant, the

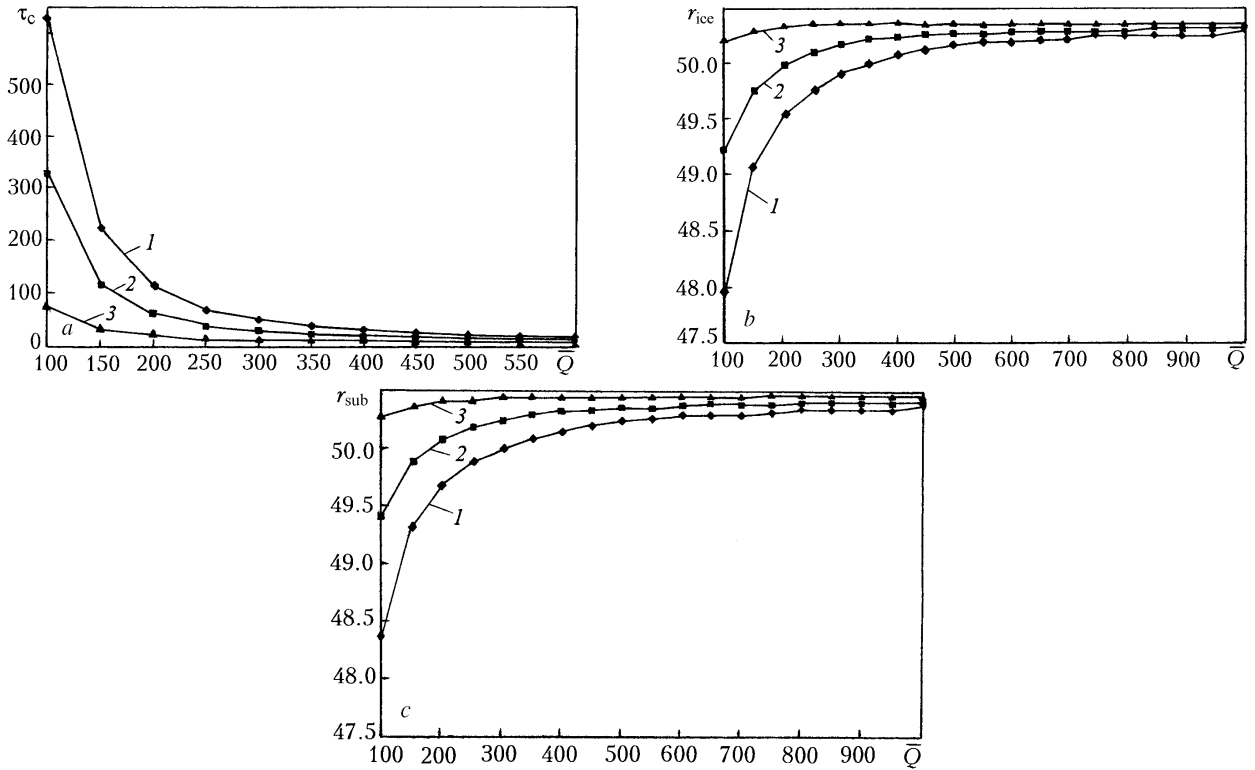


Fig. 3. Dependence of the cycle duration (a), the depth of freezing (b), and the depth of sublimation front (c) on the heat load for different effective diameters of the pores: 1)  $d = 70$ , 2) 35, and 3) 7  $\mu\text{m}$ .  $\tau_c$ , sec;  $\bar{Q}$ , W;  $r_{ice}$ , mm;  $r_{sub}$ , mm.

heat fluxes supplied to the element and removed to vacuum become equal. At this instant, the crystallization front is brought to rest, the ice filler ceases to grow, and a temporary equilibrium is established. However, the sublimation front continues to deepen, which results in a further decrease in the intensity of heat removal and in the beginning of the reverse process of melting of the ice filler. The melting front begins to rise to the external surface of the porous structure. As a result, the ice filler decreases due to two factors: ice melting from below and its sublimation from above. This continues until the ice filler disappears completely. However, in actual practice, the filler can be forced through by filtration pressure after reaching a certain critical dimension rather than disappear completely. Thereafter the liquid begins to rise quickly to the surface of the porous element. From the instant the upper boundary is reached, the entire cycle of sublimation cooling described starts again.

Figure 3a shows the dependence of the duration of the cycle  $\tau_c$  on the heat load  $\bar{Q}$  delivered to the internal surface of the porous element for different  $d$ . The cycle duration decreases drastically with increase in the thermal power from 100 to 150 W. The reason is that the rise of the heat load decreases the depth of freezing, bringing it closer to the external boundary. The difference between cycle durations for dissimilar  $d$  decreases, too. It is clear from Fig. 3a that the cycle time, especially at small heat loads, depends on the effective diameter of the pores. The cycle duration increases with  $d$ . We have  $\tau = 628$  sec for an effective diameter of the pores of  $d = 70 \mu\text{m}$ ,  $\tau = 325$  sec for  $d = 35 \mu\text{m}$ , and  $\tau = 73$  sec for  $d = 7 \mu\text{m}$  when  $\bar{Q} = 100$  W.

With increase in the heat load, the depth of freezing decreases and the ice filler is closer to the external surface, which leads to a decrease in both the cycle duration and the differences in the values of  $\tau_c$  between the porous elements with dissimilar effective diameters, since the influence of the term  $\frac{1}{1 + \xi_1/d}$  in the equation for  $j_v$  on the removed heat flux decreases due to the decrease in  $\xi_1$  in formula (6).

Analysis of the dependences of the cycle duration and the depth of freezing on the heat load  $\bar{Q}$  for the porous elements with different thermal conductivities has shown that at small  $\bar{Q}$  these values are independent, in practice, of



the effective thermal conductivity. Some influence, as the calculations demonstrate, begins to manifest itself only at large  $\bar{Q}$ . Therefore, a change in the thermophysical characteristics of the material in the presence of phase transitions weakly affects the process of sublimation cooling. This is also supported by the data of [19].

It has been shown that the largest differences in  $\tau_c$  upon change in the thickness of the porous element manifest themselves at small heat loads. With increase in the loads these differences decrease and, beginning with a certain value of  $\bar{Q}$ , remain nearly constant. Thus, the thickness of the porous element exerts the largest influence on  $\tau_c$  for small heat fluxes. The calculations show that a decrease in the thickness of the porous element leads to a reduction in the cycle duration, which is to be expected. Furthermore,  $\tau_c$  increases with porosity.

The dependences of the depth of freezing and the depth of the sublimation front on the heat load for different effective diameters of the pores  $d$  are presented in Fig. 3b and c. It is seen from Fig. 3b that the depth of freezing increases with the effective diameter, and complete freezing of the liquid in the sample is possible even with small  $\bar{Q}$ , which would be prevented. The maximum thickness of the ice layer increases with the depth of freezing. When the heat load increases, the depth of freezing decreases for all  $d$ . It is clear from Fig. 3b and c that for  $d = 7 \mu\text{m}$  the depths of freezing and sublimation are independent, in practice, of the heat load.

It also follows from the calculations that at large  $\bar{Q}$  the fronts of phase transitions are very close to each other (i.e., the thickness of the ice filler is small) and to the upper boundary  $r = R$ , and at small  $\bar{Q}$  the depth of freezing increases with decrease in the dimension of the porous element.

## CONCLUSIONS

1. The mathematical model of heat and mass transfer in a porous cylindrical layer with two phase transitions — ice–steam (ice sublimation in vacuum) and water–ice (crystallization or melting) with moving interphases has been developed and realized.

2. As a result of the realization of the mathematical model proposed, the cyclic character of heat and mass transfer in the porous cylindrical element has been revealed, which corresponds to the conclusions of [19]. The cycle duration  $\tau_c$  with small heat loads  $\bar{Q}$  substantially depends on the thickness of the porous element, the pore diameter, and the porosity, and with increase in  $\bar{Q}$  the time  $\tau_c$  is reduced and its dependence on the indicated parameters becomes weak.

3. According to the calculations (see, for example, Fig. 2), the temperature in the porous element in the process of sublimation cooling is close to  $0^\circ\text{C}$ , which correlates well with the data of [19].

4. Complete freezing of the liquid in the porous element is possible with small heat loads  $\bar{Q}$  (or with low values of the heat-flux density  $q = \bar{Q}/S$ ) and large diameters of the pores.

## NOTATION

$T$ , temperature, K;  $\varepsilon$ , porosity;  $\varepsilon^*$ , emissivity factor of nickel;  $\rho$ , density,  $\text{kg}/\text{m}^3$ ;  $c$ , specific heat,  $\text{J}/(\text{kg}\cdot\text{K})$ ;  $\lambda$ , thermal conductivity,  $\text{W}/(\text{m}\cdot\text{K})$ ;  $k$ , Boltzmann constant,  $\text{J}/\text{K}$ ;  $m$ , mass of a water molecule,  $\text{kg}$ ;  $\sigma$ , Stefan–Boltzmann constant,  $\text{W}/(\text{m}^2\cdot\text{K}^4)$ ;  $d$ , effective diameter of the pores,  $\text{m}$ ;  $Q_1$ , sublimation heat,  $\text{J}/\text{kg}$ ;  $Q_2$ , melting heat of ice,  $\text{J}/\text{kg}$ ;  $Q_3$ , heat of evaporation of water,  $\text{J}/\text{kg}$ ;  $R^*$ , gas constant of the steam,  $\text{J}/(\text{kg}\cdot\text{K})$ ;  $R$ , external radius of the porous element,  $\text{m}$ ;  $L$ , radius of the internal surface of the hollow cylinder,  $\text{m}$ ;  $D$ , thickness of the porous wall,  $\text{m}$ ;  $T_*$ , temperature on the boundary of the ice–steam phase transition, K;  $\rho_v(T^*)$ , density of the saturated vapor at  $T_*$ ,  $\text{kg}/\text{m}^3$ ;  $H$ , cylinder length,  $\text{m}$ ;  $\text{Kn}$ , Knudsen number;  $q$ , heat-flux density,  $\text{W}/\text{m}^2$ ;  $r$ , space coordinate along the cylinder radius,  $\text{m}$ ;  $Q$ , heat of phase transition,  $\text{J}/\text{kg}$ ;  $P$ , vapor pressure, Pa;  $P_0$ , pressure of the saturated vapor at  $T_0$ , Pa;  $\xi_1$ , coordinate of the sublimation front,  $\text{m}$ ;  $\xi_2$ , coordinate of the crystallization (melting) front,  $\text{m}$ ;  $j_v$ , vapor flux,  $\text{kg}/(\text{m}^2\cdot\text{sec})$ ;  $\tau$ , time,  $\text{sec}$ ;  $d^*$ , capillary diameter,  $\text{m}$ ;  $\lambda^*$ , mean free path of molecules at  $P = P_v(T_*)$ ,  $\text{m}$ ;  $\Delta\tau$ , time step,  $\text{sec}$ ;  $h$ , coordinate step,  $\text{m}$ ;  $\Delta M$ , increment in the mass as a result of melting or crystallization,  $\text{kg}$ ;  $\bar{Q}$ , heat load,  $\text{W}$ ;  $S$ , surface area of the body. Subscripts: s, skeleton; w, water; ice, ice; v, vapor; c, cycle; ph, phase; eff, effective; sub, sublimation; 1, vapor + porous body zone; 2, ice + porous body zone; 3, water + porous body zone;  $i$ , number of the step along the coordinate  $r$ ;  $k$ , number of the time step; 0, certain initial value.

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